An Extension of CreditGrades Model Approach with Lévy Processes

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Abstract

This paper proposes an extended CreditGrades model called the Lévy CreditGrades model, which is driven by a Lévy process. In this setting, quasi closed-form formulae for pricing equity options on a reference firm and for calculating its survival probabilities are derived. Moreover, using a certain Lévy CreditGrades model, we compute implied volatilities on equity options and term structures of credit default swaps (CDSs) and we examine the jump risk effects of the firm's asset value on short term CDS spreads and equity volatility skew. As a result, with this extension, our model is found to have more pregnant abilities than the original model introduced by Finger et al. [2002] and Stamicar and Finger [2005], and it is more appropriate for derivative valuation in practice.

Keywords: CreditGrades Model, Lévy Process, Equity Option, Credit Default Swap, Wiener-Hopf Factorization

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1 Introduction

This paper proposes an extended CreditGrades model for pricing equity options and CDSs simultaneously. The original CreditGrades model presented by Finger et al. [2002], and Stamicar and Finger [2005] is one of the popular approaches to link between credit and equity markets. Lévy processes are then introduced into the original model in order to describe non-continuous dynamics of reference firm's asset value. In this setting, quasi closed-form formulae for pricing equity options and for calculating survival probabilities on the firm are derived, and we focus on investigating jump effects of the firm's value on short term credit spread and equity volatility skew.

With remarkable development of derivatives such as CDSs and equity options, linkage between credit and equity markets is one of the hottest issues for practitioners. For example, capital structure arbitrage, convertible bond arbitrage, and credit relative value trading have become very popular strategies among hedge funds. Moreover, many asset managers and banks measure firm-specific credit risk in fixed-income portfolios by careful observation of the equity market, in particular, the equity option market. By incorporating the interaction between credit and equity risk, sophisticated trading strategies and risk management can be implemented.

In Merton's seminal paper [1976], a classical firm value model is introduced in order to deal with the credit risk of a specific firm, in which the company defaults if the asset value becomes less than its debt payment at maturity. Black and Cox [1976], and Leland and Toft [1996] extended Merton's model to take into account the possibility that default may happen prior to the maturity date. Besides these, there are many extensions of Merton's model; for example, stochastic interest rates (Longstaff and Schwartz [1995]), stochastic default barriers (Finger et al. [2002]), jumps in the dynamics of the firm's asset value (Zhou [1997, 2001]). These credit risk modelings are known as *structural approach*, in contrast to *intensity-based approach*.

The CreditGrades model also belongs to the class of structural approach. Although Merton's original model provides no connection to the equity option market, the CreditGrades model explicitly connects credit risk with the equity option market. There have been various empirical investigations using the model of linkage between credit and equity markets since the model was presented. For example, Veraart [2004] examined default probabilities of some commercial banks and compared the CreditGrades model with the KMV model; Bystrom [2006] investigated the predictive ability of the CreditGrades model by using empirically observed CDS spreads of iTraxx indices covering Europe. Yu [2006], Bedendo et al. [2007], and Bajlum [2007] studied capital structure arbitrage trading using the CreditGrades model.

One of the problems in the structural model approach is so-called predictability of default, which is discussed in detail in Bielecki and Rutkowski [2002], Lando [2004], and Elizalde [2005]. That is, since most of the structural models assume a continuous diffusion process for dynamics of the firm's asset value and complete information about the firm's value and the default barrier, the distance from the current firm's value to the default barrier completely inform us about the nearness of default. As a result, if the current value of the firm is far away from the barrier, both the default probability and the credit spreads in short-term are close to zero; because the process of the firm's value needs time to reach the barrier. This phenomenon contradicts empirical data in credit markets.

Another problem in the original CreditGrades model is that the implied volatility skew on equity options highly depends on the leverage ratio of the firm's financial structure. In the original CreditGrades model, because the equity process of the firm follows a sifted log-normal distribution, the equity volatility becomes the local volatility function, which depends on only the current stock price. Thus, although the CreditGrades model naturally introduces the volatility skew, it is not able to reflect unpredictable credit events into the implied volatility.

One of the approaches to overcome these problems is to include jumps in the firm's asset value process. For example, Zhou [1997, 2001] introduced an extended Merton model with jump risk, and using this model, he examined jump impacts of the firm's value. Sepp [2006] proposed two types of extended CreditGrades models with jumps and stochastic volatility respectively. However, his jump CreditGrades model dealt with only a double-exponential jump-diffusion process. Now the framework of an extended CreditGrades model with general Lévy processes is presented in this paper. Lévy processes are well-known as an appropriate class of stochastic processes with jumps in order to express various underlying asset dynamics and to price many derivative products. The processes are studied by numerous financial researchers such as Merton [1976], Barndorff-Nielsen [1997], Madan, et al. [1998], Boyarchenko and Levendorskiĭ [2002], Carr, et al. [2002], Kou [2002, 2003], Eraker, et al. [2003], Nguyen-Ngoc [2003], Sepp and Skachkov [2003], Asmussen et al. [2005], Jeannin and Pistorius [2007].

The organization of this paper is as follows: The next section describes the basic structure of our model. Section 3 introduces the fundamental formulae for equity option prices and CDS per premiums in the general CreditGrades model setting and derives the formulae for pricing equity options and calculating survival probabilities in our model, which are the main contributions in this paper. Section 4 proposes a specific model with numerical examples and shows some numerical results. The final section states the conclusion. The appendix gives mathematical tools for our model and the proofs of our formulae.

2 Models

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\leq t\leq T^*}, \mathbb{Q})$ be a filtered probability space, where T^* is some time horizon, and $\mathbb{Q}$ is a risk neutral probability measure. We consider a certain reference firm and use the following notations: S_t and V_t denote the firm's equity price per share and its asset value per share, respectively. For simplicity, it is assumed that the firm's total debt per share; denoted by B, is a strictly positive constant value.

Suppose that the asset value V_t is driven by a suitable stochastic process under the risk neutral measure $\mathbb{Q}$ and the firm defaults when the asset value hits the barrier B. Thus, the time τ of the default on time interval $(0, T]$ is defined as

$$\tau = \inf\{t \in (0, T] : V_t \leq B\}, \tag{1}$$

with τ being an $\mathbb{F}$-stopping time. In this paper, the firm's debt B is identified with the default barrier for simplicity.

Next, we propose a new model introducing a Lévy process into the original CreditGrades model. In the sequel it is called the *Lévy CreditGrades model*. Thus, under Lévy CreditGrades model approach, the firm's asset value follows

$$V_t = V_0 e^{X_t}, \tag{2}$$

where $V_0 := S_0 + B$ is the initial asset value, $X := (X_t)_{t\geq 0}$ is a one-dimensional Lévy process on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\leq t\leq T^*}, \mathbb{Q})$; i.e. X_t is adapted to $\mathcal{F}_t$, the sample paths of X are right continuous with left limits, and $X_u - X_t$ is independent of $\mathcal{F}_t$ and has the same distribution as X_{u-t} for $0 \leq t < u$. In addition, assume that the Lévy process X is exponential martingale under the risk neutral measure $\mathbb{Q}$. Note that the default time τ is a totally inaccessible stopping time because of discontinuity property of Lévy processes. This fact is very important for credit risk modeling.

We define the dynamics of the equity price as

$$S_t = \begin{cases} (V_t - B)e^{\int_0^t (r_s - d_s)ds} & \text{if} \quad t < \tau, \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

where r_t is a deterministic risk-free interest rate, and d_t is a deterministic dividend yield on the firm's equity. By virtue of the specification (3), we can estimate parameters of the asset value process V_t, which is not directly observable, by linking the asset value to the equity process and using available market data such as implied volatilities on the equity option market.

In order to analyze the Lévy CreditGrades models, we apply the characteristic function approach, which is based on the characteristic function of a Lévy process X_t. The characteristic function Ψ_{X_t} of the distribution of the random variable X_t can be represented in the following form:

$$\Psi_{X_t}(\theta) := \mathbb{E}\left[e^{i\theta X_t}\right] = \exp\left\{-t\psi_X(\theta)\right\}, \tag{4}$$

where $\mathbb{E}[\,\cdot\,]$ is the expectation operator under the risk neutral measure $\mathbb{Q}$. The function ψ_X is called the characteristic exponent of X. The following proposition gives us the explicit representation of the characteristic exponent. The proof can be found on pp.37-45 of Sato [1991].

Proposition 1 *(Lévy-Khintchine formula) Let* $X = (X_t)_{t\geq 0}$ *be a Lévy process on* $\mathbf{R}$*. Then its characteristic exponent* ψ_X *is given by*

$$\psi_X(\theta) = -i\gamma\theta + \frac{1}{2}\sigma^2\theta^2 + \int_{-\infty}^{+\infty} \left(1 - e^{i\theta x} + i\theta x \mathbf{1}_{|x|\leq 1}\right) \Pi(dx), \tag{5}$$

where $\sigma \geq 0$ *and* $\gamma \in \mathbf{R}$ *are constants, and* Π *is a measure on* $\mathbf{R} \setminus \{0\}$ *satisfying*

$$\int_{-\infty}^{+\infty} \left(1 \wedge x^2\right) \Pi(dx) < +\infty. \tag{6}$$

The parameter σ^2 is called the Gaussian coefficient and the measure Π is called the Lévy measure. The triplet (γ, σ^2, Π) is referred to as the *Lévy characteristics* of X. Intuitively, γ describes the constant drift of the process and the Gaussian coefficient σ^2 describes constant variance of the continuous component of the process. The Lévy measure Π describes the jump structure of the jump component of the process. If $\Pi = 0$, the Lévy process is Gaussian, and if $\sigma^2 = 0$, the process is a jump process without the diffusion component.

If the Lévy CreditGrades model has the following Gaussian process, we call it the *standard model*:

$$X_t = \sigma W_t - \frac{1}{2}\sigma^2 t, \tag{7}$$

where W_t is a one-dimensional standard Brownian motion under the risk neutral measure $\mathbb{Q}$, and σ is asset volatility. In the case of the standard model, since the equity price process (3) is a shifted log-normal process, the equity volatility σ_t^S becomes the following local volatility function:

$$\sigma_t^S = \sigma \frac{S_t + B}{S_t}. \tag{8}$$

Therefore, the standard model can describe the implied equity volatility skew naturally. However, because it strongly depends on the leverage ratio of the firm's debt; the standard model of different firms describe exactly the same volatility skew shapes when the firms have the same debt-equity ratio and asset volatility, and the volatility skew cannot reflect rare credit events in the future, which might damage the firm's value. Furthermore, the standard model cannot describe higher shorter credit spreads of CDS; because the process (7) is continuous. The idea of making higher shorter spreads is to set a stochastic default barrier with some distribution. However it is difficult to choose the appropriate distribution of the stochastic default barrier, because the stochastic behavior of the barrier is usually unobservable.

On the other hand, by introducing a Lévy process into the model, our models can describe the jump risk of the firm's asset value and the default event becomes unpredictable without a stochastic default barrier. Therefore, not only can the Lévy CreditGrades models generate higher shorter credit spreads of CDS; but also it can draw the volatility skew, including speculation of rare event risks.

3 Pricing Equity Options and CDSs

In this section, how to price equity options and CDSs of a certain firm under the Lévy CreditGrades model are considered; also the formulae of the equity option prices and the survival probabilities are derived. These formulae, which are quasi closed-form solutions, are the main contributions of this paper.

3.1 Fundamental formulae of Equity Options and CDSs

We first provide the fundamental formula of equity option pricing in the general CreditGrades model approach. In this approach, equity options are evaluated as down and out options, which vanish after the firm's asset value hits the default barrier. Therefore the payoff function of the call option with strike K and maturity T is defined by

$$\left(S_T - K\right)^+ \mathbf{1}_{\{\tau > T\}}, \tag{9}$$

where τ is the default time. Then, the call option price at the initial time is given by

$$C = \mathbb{E}\left[e^{-\int_0^T r_t dt}\left(S_T - K\right)^+ \mathbf{1}_{\{\tau > T\}}\right]. \tag{10}$$

Next, we provide the fundamental formula of CDS per premiums, and it is shown that survival probabilities of the firm denoted by $\mathbb{Q}\left(\tau > t\right)$ allow us to price CDS per premium. Indeed, the fixed leg of a CDS can be represented as follows:

$$\begin{aligned} \text{Fixed Leg} &= \mathbb{E}\left[\int_0^T e^{-\int_0^t r_u du} cN\mathbf{1}_{\{\tau > t\}} dt\right] \\ &= cN \int_0^T e^{-\int_0^t r_u du}\mathbb{Q}\left(\tau > t\right) dt, \end{aligned} \tag{11}$$

where c is the premium of the CDS contract with maturity T, and N is the notional amount of the contract. On the other hand, the floating leg can be represented as follows:

$$\begin{aligned} \text{Floating Leg} &= \mathbb{E}\left[\int_0^T e^{-\int_0^t r_u du}(1-R)N\mathbf{1}_{\{\tau \in dt\}} dt\right] \\ &= (1-R)N \\ &\times \left(1 - e^{-\int_0^T r_t dt}\mathbb{Q}(\tau > T) - \int_0^T r_t e^{-\int_0^t r_u du}\mathbb{Q}(\tau > t) dt\right), \end{aligned} \tag{12}$$

where R is a constant recovery rate of the firm. The CDS per premium is chosen to equate the fixed leg and the floating leg, thus it can be calculated by

$$
c = (1-R)\frac{1 - e^{-\int_0^T r_t dt}\mathbb{Q}(\tau > T) - \displaystyle\int_0^T r_t e^{-\int_0^t r_u du}\mathbb{Q}(\tau > t)dt}{\displaystyle\int_0^T e^{-\int_0^t r_u du}\mathbb{Q}\left(\tau > t\right)dt}. \tag{13}
$$

Note that if the survival probabilities of the firm under the risk neutral measure $\mathbb{Q}$ is obtained, the CDS per premiums can be calculated. Hence, it is sufficient for pricing the premiums to know how to calculate the survival probabilities.

3.2 Equity Option Prices and Survival Probabilities under the Standard Model

In this subsection, we provide the formulae of equity option prices and survival probabilities under the standard model. Although these formulae, which are introduced by Finger et al. [2002], are simple; they play important roles for the Lévy CreditGrades model. In following subsection, we apply the formulae for robust calculation of both equity option prices and survival probabilities under our model.

The call option of the standard model is evaluated as a down and out call option with a zero knock-out barrier under a shifted log-normal equity process. Thus, the call price with an asset volatility σ at the initial time, which is denoted by C^{σ}, is given by

$$
\begin{aligned}
C^{\sigma} =& C_{\mathrm{BS}}(T, S_0 + B, K + B, \bar{r}, \bar{d}) \\
&- \frac{S_0 + B}{B} C_{\mathrm{BS}}(T, \frac{B^2}{S_0 + B}, K + B, \bar{r}, \bar{d}), \qquad (14) \\
\text{where} \quad & \bar{r} = \frac{1}{T}\int_0^T r_t dt, \quad \bar{d} = \frac{1}{T}\int_0^T d_t dt,
\end{aligned}
$$

$C_{\mathrm{BS}}(T, S, K, r, d)$ is the Black-Scholes price of a call option with maturity T, strike K, constant interest rate r, and constant dividend rate d on underlying price S with volatility σ.

The survival probability of the standard model with an asset volatility σ, which is denoted by $\mathbb{Q}(\tau > T; \sigma)$, can be calculated by the following formula:

$$\begin{aligned}\mathbb{Q}(\tau > T; \sigma) = \mathcal{N}&\left(\frac{\log\left(\frac{S_0+B}{B}\right) - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right) \\ &- \frac{S_0+B}{B}\mathcal{N}\left(\frac{\log\left(\frac{B}{S_0+B}\right) - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right),\end{aligned} \tag{15}$$

where $\mathcal{N}(\cdot)$ is the cumulative distribution function of standard normal distribution.

3.3 Equity Option Prices and Survival Probabilities under the Lévy CreditGrades Model

Before we derive the formulae of the equity option prices and survival probabilities under the Lévy CreditGrades models, we introduce the Wiener-Hopf factors of a Lévy process, which are helpful in evaluating the Fourier transforms of quantities related to maximum and minimum of a Lévy process. First, the maximum and minimum processes associated with a Lévy process X_t are defined as

$$M_t := \max_{0 \le s \le t} X_s, \qquad N_t := \min_{0 \le s \le t} X_s \tag{16}$$

In the following proposition, the Wiener-Hopf factors associated with the process X_t are introduced. The proof of Proposition 2 can be found on p.334 of Sato [1991].

Proposition 2 *Let $q > 0$. There exist a unique pair of characteristic functions $\Phi^+_{q,X}(\theta)$ and $\Phi^-_{q,X}(\theta)$ of infinitely divisible distributions having zero drift and supported on $[0,\infty)$ and $(-\infty,0]$ respectively such that*

$$\frac{q}{q-\psi_X(\theta)} = \Phi^+_{q,X}(\theta)\Phi^-_{q,X}(\theta), \quad \theta \in \mathbf{R}. \tag{17}$$

These functions have the following representations.

$$\Phi^+_{q,X}(\theta) := \exp\left\{\int_0^{+\infty} t^{-1}e^{-qt}dt \int_0^{+\infty} \left(e^{i\theta x}-1\right)dF_{X_t}(x)\right\}, \tag{18}$$

$$\Phi^-_{q,X}(\theta) := \exp\left\{\int_0^{+\infty} t^{-1}e^{-qt}dt \int_{-\infty}^{0} \left(e^{i\theta x}-1\right)dF_{X_t}(x)\right\}, \tag{19}$$

where $F_{X_t}(\cdot)$ is the distribution function of a random variable X_t.

The function $\Phi^+_{q,X}(\theta)$ and $\Phi^-_{q,X}(\theta)$ are called the *Wiener-Hopf factors*. The function $\Phi^+_{q,X}(\theta)$ can be continuously extended to a bounded analytic function without zeros on the upper half plane and $\Phi^-_{q,X}(\theta)$ can be similarly extended to the lower half plane.

The following theorem shows the equity option pricing formula under the Lévy CreditGrades model. The Wiener-Hopf factors play crucial roles for deriving the pricing formulae. The proof of Theorem 1 is provided in appendix B.

Theorem 1 *(Option pricing formula under the Lévy CreditGrades model) Let X_t be a Lévy process driving the CreditGrades model, and $\alpha, \beta > 0, \gamma \in \mathbf{R}$ and $\sigma > 0$ be some real values. Then the equity option price C with strike K and maturity T is given by the following representation:*

$$C = e^{-\int_0^T d_t dt}(S_0+B)f(T,k,b) + C^\sigma, \tag{20}$$

where

$$
\begin{aligned}
f(T,k,b) &:= \frac{e^{-(\alpha k+\beta b-\gamma T)}}{(2\pi)^3}\iiint_{\mathbf{R}^3} e^{-i(uk+vb-qT)}\kappa(\gamma+iq,u,v)dudvdq,\\
\kappa(q,u,v) &:= \frac{1}{q(iu+\alpha)(iv+\beta)(iu+\alpha+1)}\\
&\quad\times\Big\{\Phi^+_{q,X}(u-i[\alpha+1])\Phi^-_{q,X}(u-i[\alpha+\beta+1])\\
&\qquad -\Phi^+_{q,Y}(u-i[\alpha+1])\Phi^-_{q,Y}(u-i[\alpha+\beta+1])\Big\},\\
k &:= \log\left(\frac{Be^{\int_0^T(r_t-d_t)dt}+K}{(S_0+B)e^{\int_0^T(r_t-d_t)dt}}\right),\\
b &:= \log\left(\frac{B}{S_0+B}\right),
\end{aligned}
\tag{21}
$$

$\Phi^{\pm}_{q,X}(\cdot)$ *and* $\Phi^{\pm}_{q,Y}(\cdot)$ *denote the Wiener-Hopf factors of the Lévy process* X_t *and a Gaussian process* $Y_t := \sigma W_t - \frac{1}{2}\sigma^2 t$ *respectively.*

Next, we derive the survival probability formula under the Lévy CreditGrades model. In the following theorem, the Wiener-Hopf factors again play crucial roles for calculating survival probabilities. The proof of Theorem 2 is provided in appendix C.

Theorem 2 *(Survival probability formula under the Lévy CreditGrades models) Let* X_t *be a Lévy process driving a CreditGrades model, and* $\alpha > 0, \gamma \in \mathbf{R}$ *and* $\sigma > 0$ *be some real values. Then the survival probability* $\mathbb{Q}(\tau > t)$ *is given by the following representation:*

$$
\mathbb{Q}(\tau > t) = g(t,b) + \mathbb{Q}(\tau > t;\sigma), \tag{22}
$$

where

$$\begin{aligned} g(t,b) &:= \frac{e^{-(\alpha b-\gamma t)}}{(2\pi)^2}\iint_{\mathbf{R}^2} e^{-i(ub-qt)}\xi(\gamma+iq,u)dudq, \\ \xi(q,u) &:= \frac{\Phi^-_{q,X}(u-i\alpha)-\Phi^-_{q,Y}(u-i\alpha)}{q(iu+\alpha)}, \\ b &:= \log\left(\frac{B}{S_0+B}\right), \end{aligned} \tag{23}$$

$\Phi^-_{q,X}(\cdot)$ and $\Phi^-_{q,Y}(\cdot)$ denote the Wiener-Hopf factors of the Lévy process X_t and a Gaussian process $Y_t := \sigma W_t - \frac{1}{2}\sigma^2 t$ respectively.

By using the parameter α and β in Theorem 1 and 2, singularity on the integrands in the Fourier inversion can be avoided, since the numerical computation method, such as the *fast Fourier transform method*, evaluates the integrands at $u = 0$ and $v = 0$. The parameter γ is used for changing the inverse Laplace transform, which has some difficult problems in the numerical computation, into the Fourier transform. Moreover, in Theorem 1, by considering the difference between option prices of the Lévy CreditGrades model and the standard model, the Fourier inversion converges quickly at infinity. The same technique is applied for Theorem 2. See pp.361-363 in Cont and Tankov (2003) for details.

Note that in order to compute option prices and survival probabilities, we need to derive the explicit expression of the Wiener-Hopf factors of the Lévy process X_t driving the model. However, in general, it is difficult to find the explicit expression of the factors. Therefore, in the following section, a certain tractable class of Lévy processes for numerical examples are introduced.

4 A Concrete Example

In this section, we show numerical examples of equity option prices and CDS premiums through a certain Lévy CreditGrades model. According to Theorem

1 and 2, in order to price them, we have to obtain the Wiener-Hopf factors of the Lévy process X_t driving the model. However, in general, it is difficult to find explicit forms of the factors and we must evaluate them numerically. Computations using the equation (18) and (19) are not very efficient, because these equations involve the probability density function of X_t which is usually not available in closed form.

Boyarchenko and Levendorskiĭ [2002] give a more efficient expression which is valid for tempered stable, normal inverse Gaussian and several other Lévy processes:

$$\Phi_{q,X}^{+}(\theta) = \exp\left\{ \frac{\theta}{2\pi i} \int_{-\infty+i\omega}^{+\infty+i\omega} \frac{\log\left(q+\psi_X(\xi)\right)}{\xi\left(\theta-\xi\right)} d\xi \right\}, \tag{24}$$

with some $\omega < 0$ such that $\Phi_{q,X}^{+}(\theta)$ is analytic in the half plane $\Re\theta > \omega$. Note that the above integral must be computed for all values of θ and q to obtain a equity option price and a survival probability, and these computations are still time-consuming.

On the other hand, we choose a certain class of Lévy processes called *spectrally negative Lévy processes* for our model. Spectrally negative Lévy processes have only negative jumps, i.e. the Lévy measure Π of X_t satisfies that $\Pi\left((0,+\infty)\right) = 0$. Using spectrally negative Lévy processes, we can express negative jumps of the firm's asset value with causes of some credit events, e.g. accounting practices, a scandal concerning the executives, detection of defective products, and other bad news. If X_t is a spectrally negative Lévy process, the Wiener-Hopf factors are given by

$$\begin{aligned} \Phi_{q,X}^{+}(\theta) &= \frac{\eta_q}{\eta_q - i\theta}, \\ \Phi_{q,X}^{-}(\theta) &= \frac{q\left(\eta_q - i\theta\right)}{\eta_q\left(q-\psi_X(\theta)\right)}, \end{aligned} \tag{25}$$

where η_q is the unique real root of $q + \psi_X(-i\eta_q)$. See pp.346-348 of Sato [1991] for details.

In the case of the standard model with a Gaussian process $Y_t := \sigma W_t - \frac{1}{2}\sigma^2 t$, its characteristic exponent is $\psi_Y(\theta) = -\frac{1}{2}\sigma^2\left(\theta^2 + i\theta\right)$. Since the process belongs to the class of spectrally negative Lévy processes, the Wiener-Hopf factors $\Phi^+_{q,Y}(\theta)$ and $\Phi^-_{q,Y}(\theta)$ can be obtained as the following expressions:

$$\begin{aligned} \Phi^+_{q,Y}(\theta) &= \frac{\eta_+}{\eta_+ + i\theta}, \quad \text{where} \quad \eta_+ = -\frac{1}{2} + \frac{1}{\sigma}\sqrt{\frac{\sigma^2}{4} + 2q}, \\ \Phi^-_{q,Y}(\theta) &= \frac{\eta_-}{\eta_- - i\theta}, \quad \text{where} \quad \eta_- = +\frac{1}{2} + \frac{1}{\sigma}\sqrt{\frac{\sigma^2}{4} + 2q}. \end{aligned} \tag{26}$$

Note that the equations (26) can be used for calculating (21) in Theorem 1 and (23) in Theorem 2.

4.1 Model Specification

Let us specify a certain jump-diffusion process driving the Lévy CreditGrades model as follows:

$$X_t = \mu t + \sigma W_t - \sum_{j=1}^{N_t} Y_j, \tag{27}$$

where $\sigma > 0$, N_t and W_t denote Poisson process with intensity λ and Brownian motion respectively under the risk-neutral measure $\mathbb{Q}$, and the sequence of jump sizes $(Y_j)_{j\in\mathbf{N}}$ are i.i.d. random variables according to exponential distribution with parameter a. We assume that the process X_t satisfies exponential martingale under the measure $\mathbb{Q}$. Note that the process X_t is a spectrally negative Lévy process and if the jump intensity $\lambda = 0$, this model is equivalent to the standard model.

First, we derive the characteristic exponent of X_t. Let $Z_t := \sigma W_t - \sum_{j=1}^{N_t} Y_j$, which is the random part of X_t. By Lévy-Khintchine formula, the characteristic exponent of Z_t is given by

$$\begin{aligned} \psi_Z(\theta) &= \frac{1}{2}\sigma^2\theta^2 + \lambda\left(1 - \Psi_Y(-\theta)\right) \\ &= \frac{1}{2}\sigma^2\theta^2 + \lambda\left(1 - \frac{a}{a+i\theta}\right), \end{aligned} \tag{28}$$

where $\Psi_Y(\theta) := a/(a - i\theta)$ is the characteristic function of the jump size Y_j. Because the process X_t satisfies exponential martingale under the risk-neutral measure $\mathbb{Q}$; its drift μ, which is called *convexity correction*, must be

$$\mu = -\psi_Z(-i) = -\frac{1}{2}\sigma^2 + \frac{\lambda}{a+1}. \tag{29}$$

Therefore, the characteristic exponent of the process (27) is given by

$$\psi_X(\theta) = -i\mu\theta + \frac{1}{2}\sigma^2\theta^2 + \lambda\left(1 - \frac{a}{a+i\theta}\right). \tag{30}$$

Next, in order to obtain the Wiener-Hopf factors of X_t, the equation $q + \psi_X(-i\eta_q) = 0$ is solved in terms of η_q. This equation can be rewritten as

$$\sigma^3\eta_q^3 + (a\sigma^2 + 2\mu)\eta_q^2 + 2(a\mu - \lambda + q)\eta_q + 2aq = 0. \tag{31}$$

Since the equation (31) is a third degree polynomial equation, we can apply Cardano formula to solve it, i.e. the unique real solution η_q is given by

$$
\begin{aligned}
\eta_q = & -\frac{a_2}{3a_1} + \sqrt[3]{-\frac{a_2^3}{27a_1^3} + \frac{a_2 a_3}{6a_1^2} - \frac{a_4}{2a_1} + \frac{1}{6}\sqrt{\frac{D}{3}}} \\
& + \sqrt[3]{-\frac{a_2^3}{27a_1^3} + \frac{a_2 a_3}{6a_1^2} - \frac{a_4}{2a_1} - \frac{1}{6}\sqrt{\frac{D}{3}}},
\end{aligned} \tag{32}
$$

where $a_1 = \sigma^3, \quad a_2 = a\sigma^2 + 2\mu, \quad a_3 = 2(a\mu - \lambda + q), \quad a_4 = 2aq,$

$$
D = 4\left(-\frac{a_2^2}{3a_1^2} + \frac{a_3}{a_1}\right)^3 + 27\left(\frac{2a_2^3}{27a_1^3} - \frac{a_2 a_3}{3a_1^2} + \frac{a_4}{a_1}\right)^2.
$$

Substituting (32) for (25), we obtain the Wiener-Hopf factors of the process X_t for all q.

4.2 Numerical Results

Using the model (27), this subsection shows the numerical results of equity option prices and CDS per premiums. Note that the model has only three parameters, i.e. σ, λ and a. In addition, it is necessary to set an initial stock price S_0, a constant total debt B, deterministic interest rate r_t, and deterministic dividend yield d_t. Let us suppose that $S_0 = 100$, $B = 100$, $\sigma = 0.2$, $a = 10$ and $r_t = d_t = 0$ for all $t \geq 0$. Moreover, in order to examine the jump effects, we set four values on the jump-intensity parameter; i.e. $\lambda = 0.00$, 0.25, 0.50, or 1.00.

First, we compute equity option prices with different strikes and maturities. Table 1, 2 and 3 show equity call option prices with 3-month, 6-month and 1-year maturities respectively. We can confirm that the jump effects on the model force up the option prices. Figure 1, 2 and 3 plot their implied equity volatilities. Note that all of the implied volatility smiles are asymmetrical so-called volatility skews. The greater jump risk there is in the model, the steeper the slope of the volatility skews is. Furthermore, the volatility skews with longer maturity tend to be flatten by the central limit theorem on the Lévy process. As for the numerical results of the option pricing, we can say that even if two firms have the same debt-equity ratio, different volatility skew curves of the firms can be drawn by using the Lévy CreditGrades models with different parameters. Hence, the Lévy CreditGrades models have the rich ability of drawing implied volatility skews and are more suitable for observed option markets than the original models.

Table 1: Equity Call Option Prices with 3-Month Maturity

Moneyness (K/S_0)	0.6	0.8	1.0	1.2	1.4
$\lambda = 0.00$	40.080	21.425	7.976	1.908	0.295
$\lambda = 0.25$	40.232	21.730	8.278	2.041	0.329
$\lambda = 0.50$	40.373	22.021	8.561	2.170	0.353
$\lambda = 1.00$	40.660	22.594	9.124	2.454	0.430

Table 2: Equity Call Option Prices with 6-Month Maturity

Moneyness (K/S_0)	0.6	0.8	1.0	1.2	1.4
$\lambda = 0.00$	40.618	23.545	11.274	4.422	1.441
$\lambda = 0.25$	40.933	24.031	11.740	4.727	1.584
$\lambda = 0.50$	41.235	24.504	12.202	5.040	1.741
$\lambda = 1.00$	41.805	25.402	13.118	5.679	2.067

Table 3: Equity Call Option Prices with 1-Year Maturity

Moneyness (K/S_0)	0.6	0.8	1.0	1.2	1.4
$\lambda = 0.00$	42.372	27.178	15.931	8.584	4.295
$\lambda = 0.25$	42.935	27.910	16.654	9.170	4.698
$\lambda = 0.50$	43.476	28.616	17.364	9.759	5.118
$\lambda = 1.00$	44.498	29.957	18.748	10.938	5.982

Next, we compute CDS per premiums with the recovery rate $R = 0.40$. Table 4 shows CDS per premium for each maturity. Moreover Table 5 shows the decay ratios of CDS premiums for each period by number of years. That is,

Figure 1: Implied Volatilities on the 3-Month Options

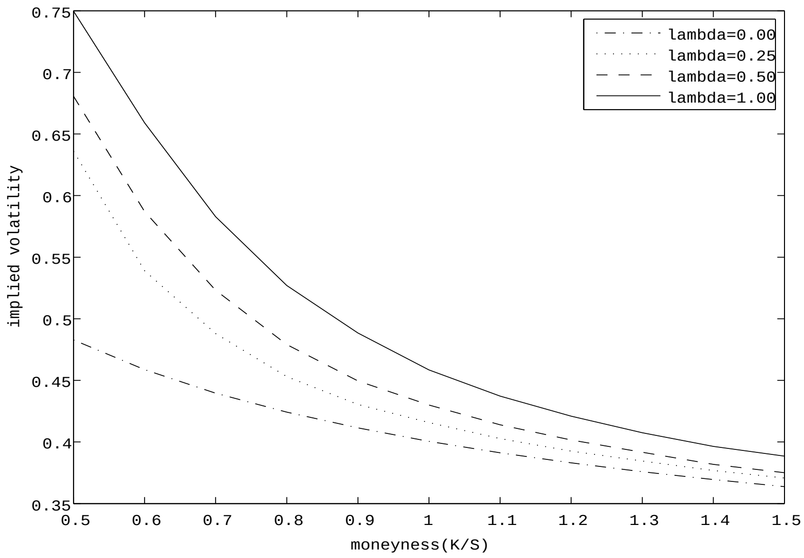


Figure 2: Implied Volatilities on the 6-Month Options

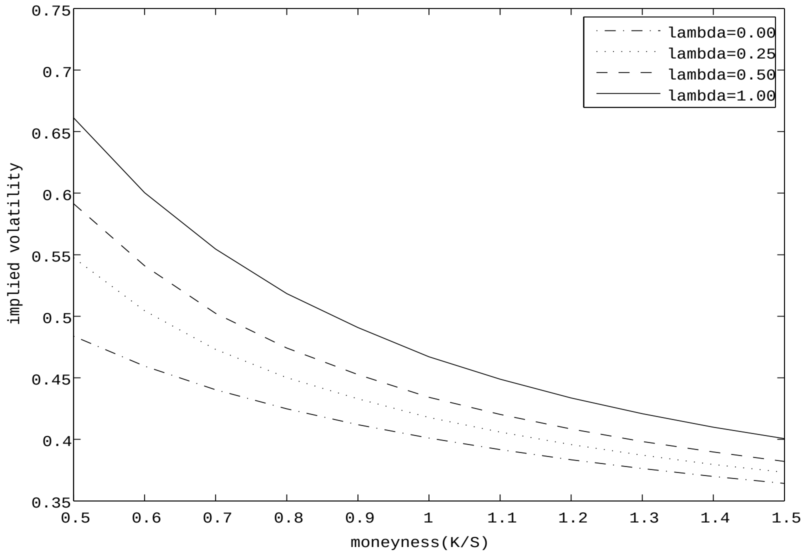
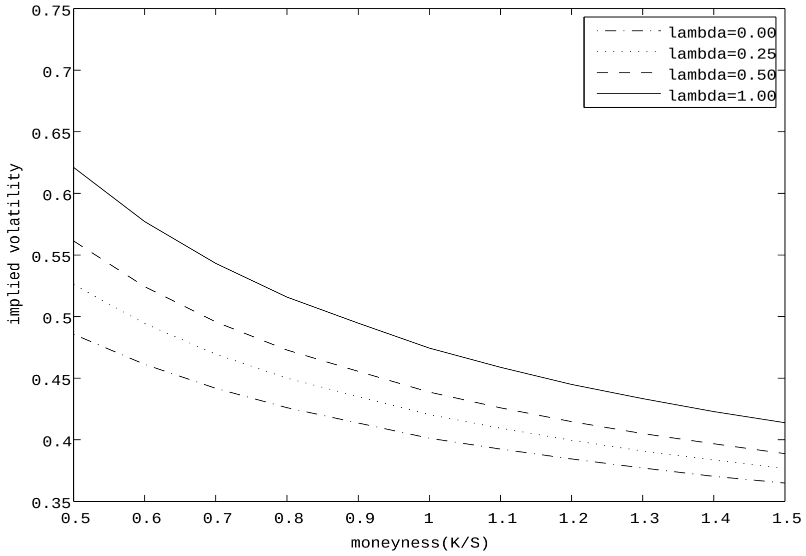


Figure 3: Implied Volatilities on the 1-Year Options



the ratio of n to $(n-1)$ year is defined as follows:

$$\text{decay ratio} := \frac{n\text{-year CDS per premium} - (n-1)\text{-year CDS per premium}}{n\text{-year CDS per premium}} (\%)$$

Note that the existence of jump risk generates higher short-term spreads, which is in good agreement with empirical observation. On the other hand, when the standard model (in the case of $\lambda = 0.00$) is used, short-term spreads decrease very rapidly and seem to converge to zero under 1-year maturity. As with the numerical results of CDS pricing, we find that the Lévy CreditGrades models are able to generate higher short-term spreads without a stochastic default barrier.

5 Conclusion

In this paper, we propose an extended CreditGrades model called the Lévy CreditGrades model, which is driven by a Lévy process. Our main contribu-

Table 4: CDS per Premiums (bp)

Time to Maturity	1-Year	2-Year	3-Year	4-Year	5-Year
$\lambda = 0.00$	6	59	126	185	209
$\lambda = 0.25$	24	95	169	221	252
$\lambda = 0.50$	45	136	210	261	293
$\lambda = 1.00$	96	212	289	331	347

Table 5: Decay Ratio of CDS per Premiums

Period	2 to 1-Year	3 to 2-Year	4 to 3-Year	5 to 4-Year
$\lambda = 0.00$	89.97%	53.12%	31.70%	11.66%
$\lambda = 0.25$	74.71%	43.49%	23.56%	12.27%
$\lambda = 0.50$	67.29%	35.03%	19.60%	10.97%
$\lambda = 1.00$	54.60%	26.88%	12.65%	4.48%

tion is to derive the pricing formulae of equity options and CDSs under this model. Moreover, for a numerical example, we provide a specific Lévy Credit-Grades model introducing a spectrally negative Lévy process, then we show a concrete calculation scheme in this model. As numerical results, we find that our models have the rich representation of equity option and CDS pricing and are appropriate in practice.

A Mathematical Tools

Mathematical tools for the proof of Theorem 1 and 2 are introduced below. The first two lemmas show that the Wiener-Hopf factors can be used for computing quantities related to the maximum and minimum of a Lévy process. The proof of the lemmas can be found on p.341 of Sato [1991].

Lemma 1 *(Wiener-Hopf factorization for a maximum process) The Laplace transform in t of the joint characteristic function of $(M_t, X_t - M_t)$ is given by*

$$q \int_0^{+\infty} e^{-qt} \mathbb{E}\left[e^{ixM_t + iy(X_t - M_t)}\right] dt = \Phi^+_{q,X}(x) \Phi^-_{q,X}(y), \tag{33}$$

for any $q > 0$ and $x,\ y \in \mathbf{R}$.

Lemma 2 *(Wiener-Hopf factorization for a minimum process) The Laplace transform in t of the joint characteristic function of $(N_t, X_t - N_t)$ is given by*

$$q \int_0^{+\infty} e^{-qt} \mathbb{E}\left[e^{ixN_t + iy(X_t - N_t)}\right] dt = \Phi^+_{q,X}(y)\Phi^-_{q,X}(x), \tag{34}$$

for any $q > 0$ and $x,\ y \in \mathbf{R}$.

Because it is difficult to compute the inverse Laplace transform numerically, by the following lemma, we change the inverse Laplace transform into the Fourier transform.

Lemma 3 *Let γ be a constant number and $f(t)$ be the inverse Laplace transform of $\bar{f}(\kappa)$:*

$$f(t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{\kappa t} \bar{f}(\kappa) d\kappa. \tag{35}$$

Then $f(t)$ is expressed as the following Fourier transform.

$$f(t) = \frac{e^{\gamma t}}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega t} \bar{f}(\gamma + i\omega) d\omega. \tag{36}$$

Proof of Lemma 3: By changing the parameter κ into $\kappa = \gamma + i\omega$, then substituting this parameter for (35), the expression (36) can be obtained.

□

B Proof of Theorem 1

The payoff function (9) can be expressed as follows:

$$
\begin{aligned}
(S_T - K)^+ \mathbf{1}_{\{\tau > T\}} &= \left((V_T - B)\, e^{\int_0^T (r_s - d_s) ds} - K \right)^+ \mathbf{1}_{\left\{\min_{0 \le s \le T} V_s > B\right\}} \\
&= \left(\tilde{V}_0 e^{X_T} - \tilde{K} \right)^+ \mathbf{1}_{\left\{\min_{0 \le s \le T} X_s > b\right\}},
\end{aligned}
\tag{37}
$$

where $\tilde{V}_0 := V_0 e^{\int_0^T (r_s - d_s) ds}$, $\tilde{K} := B e^{\int_0^T (r_s - d_s) ds} + K$, and $b := \log\left(B/(S_0 + B)\right)$. Thus the call price of the Lévy CreditGrades model with a Lévy process X_t is given by

$$
\begin{aligned}
C &= \mathbb{E}\left[e^{-\int_0^T r_t dt} (S_T - K)^+ \mathbf{1}_{\{\tau > T\}} \right] \\
&= \mathbb{E}\left[e^{-\int_0^T r_t dt} \left(\tilde{V}_0 e^{X_T} - \tilde{K} \right)^+ \mathbf{1}_{\left\{\min_{0 \le s \le T} X_s > b\right\}} \right] \\
&= e^{-\int_0^T r_t dt} \tilde{V}_0\, \mathbb{E}\left[\left(e^{X_T} - e^k \right)^+ \mathbf{1}_{\left\{N_T^X > b\right\}} \right],
\end{aligned}
\tag{38}
$$

where $k := \log(\tilde{K}/\tilde{V}_0)$ and $N_T^X := \min_{0 \le s \le T} X_s$. Similarly, the call price of the standard model with a Gaussian process $Y_t := \sigma W_t - \frac{1}{2}\sigma^2 t$ is given by

$$
C^\sigma = e^{-\int_0^T r_t dt} \tilde{V}_0\, \mathbb{E}\left[\left(e^{Y_T} - e^k \right)^+ \mathbf{1}_{\left\{N_T^Y > b\right\}} \right],
\tag{39}
$$

where $N_T^Y := \min_{0 \le s \le T} Y_s$.

We concentrate on calculating the difference between the call price of the Lévy CreditGrades model and that of the standard model. To do this, we define the following function:

$$
\begin{aligned}
f(T, k, b) &:= \frac{C - C^\sigma}{e^{-\int_0^T r_t dt} \tilde{V}_0} \\
&= \mathbb{E}\left[\left(e^{X_T} - e^k \right)^+ \mathbf{1}_{\left\{N_T^X > b\right\}} \right] - \mathbb{E}\left[\left(e^{Y_T} - e^k \right)^+ \mathbf{1}_{\left\{N_T^Y > b\right\}} \right].
\end{aligned}
\tag{40}
$$

Then we consider the double Fourier transform of the function $e^{\alpha k + \beta b} f(T, k, b)$:

$$\begin{aligned}
&\iint_{\mathbf{R}^2} e^{iuk+ivb} e^{\alpha k+\beta b} f(T,k,b)\,dk db \\
&= \iint_{\mathbf{R}^2} dk db \iint_{\mathbf{R}^2} dx dy\, e^{(iu+\alpha)k+(iv+\beta)b} \\
&\times \Big\{ \left(e^x - e^k\right)^+ \mathbf{1}_{\{y\,>\,b\}} \rho_{X_T,N_T^X}(x,y) - \left(e^x - e^k\right)^+ \mathbf{1}_{\{y\,>\,b\}} \rho_{Y_T,N_T^Y}(x,y) \Big\} \\
&= \iint_{\mathbf{R}^2} dx dy\, \rho_{X_T,N_T^X}(x,y) \int_{-\infty}^{y} \int_{-\infty}^{x} e^{(iu+\alpha)k+(iv+\beta)b} \left(e^x - e^k\right) dk db \\
&\qquad - \iint_{\mathbf{R}^2} dx dy\, \rho_{Y_T,N_T^Y}(x,y) \int_{-\infty}^{y} \int_{-\infty}^{x} e^{(iu+\alpha)k+(iv+\beta)b} \left(e^x - e^k\right) dk db \qquad (41) \\
&= \frac{1}{(iu+\alpha)(iv+\beta)(iu+\alpha+1)} \\
&\times \Big\{ \iint_{\mathbf{R}^2} e^{(iu+\alpha+1)x+(iv+\beta)y} \rho_{X_T,N_T^X}(x,y) dx dy \\
&\qquad - \iint_{\mathbf{R}^2} e^{(iu+\alpha+1)x+(iv+\beta)y} \rho_{Y_T,N_T^Y}(x,y) dx dy \Big\} \\
&= \frac{\Psi_{X_T,N_T^X}(u-i\alpha-i, v-i\beta) - \Psi_{Y_T,N_T^Y}(u-i\alpha-i, v-i\beta)}{(iu+\alpha)(iv+\beta)(iu+\alpha+1)},
\end{aligned}$$

where $\rho_{X,Z}(\cdot,\cdot)$ and $\Psi_{X,Z}(\cdot,\cdot)$ denote the joint density function and the joint characteristic function of the random vector (X, Z) respectively.

Let the function $\kappa(q,u,v)$ denote the Laplace transform in T of (41):

$$\kappa(q,u,v) := \int_0^{+\infty} e^{-qT} \iint_{\mathbf{R}^2} e^{iuk+ivb} e^{\alpha k+\beta b} f(T,k,b)\,dk db dT. \qquad (42)$$

By using Lemma 2, the function $\kappa(q,u,v)$ can be expressed as

$$
\begin{aligned}
\kappa(q,u,v) &= \frac{1}{(iu+\alpha)(iv+\beta)(iu+\alpha+1)} \\
&\quad \times \left\{ \int_0^{+\infty} e^{-qT} \Psi_{X_T, N_T^X}(u - i\alpha - i, v - i\beta) dT \right. \\
&\qquad \left. - \int_0^{+\infty} e^{-qT} \Psi_{Y_T, N_T^Y}(u - i\alpha - i, v - i\beta) dT \right\} \\
&= \frac{1}{q(iu+\alpha)(iv+\beta)(iu+\alpha+1)} \\
&\quad \times \left\{ \Phi^+_{q,X}(u - i[\alpha+1]) \Phi^-_{q,X}(u + v - i[\alpha+\beta+1]) \right. \\
&\qquad \left. - \Phi^+_{q,Y}(u - i[\alpha+1]) \Phi^-_{q,Y}(u + v - i[\alpha+\beta+1]) \right\},
\end{aligned}
\tag{43}
$$

Thus, by inverting the double Fourier transform and the Laplace transform, the function $e^{\alpha k + \beta b} f(T,k,b)$ can be obtained:

$$
e^{\alpha k + \beta b} f(T,k,b) = \frac{1}{2\pi i} \int_{\varsigma - i\infty}^{\varsigma + i\infty} e^{qT} \frac{1}{(2\pi)^2} \iint_{\mathbf{R}^2} e^{-iuk - ivb} \kappa(q,u,v) du dv dq. \tag{44}
$$

Applying Lemma 3 for (44), we complete the proof of Theorem 1.

□

C Proof of Theorem 2

By definition of the default time τ, the survival probability $\mathbb{Q}(\tau > t)$ can be expressed as follows:

$$
\begin{aligned}
\mathbb{Q}(\tau > t) &= \mathbb{Q}\left(\min_{0 \le s \le t} V_s > B \right) \\
&= \mathbb{Q}\left((S_0 + B) \exp\left\{ \min_{0 \le s \le t} X_s \right\} > B \right) \\
&= \mathbb{Q}\left(N_t^X > b \right) = \mathbb{E}\left[\mathbf{1}_{\{N_t^X > b\}} \right],
\end{aligned}
\tag{45}
$$

where $b := \log(B/(S_0+B))$ and $N_t^X := \min_{0\leq s\leq t} X_s$. Similarly, under a standard model with a Gaussian process $Y_t := \sigma W_t - \frac{1}{2}\sigma^2 t$, its survival probability $\mathbb{Q}(\tau > t; \sigma)$ is given by

$$\mathbb{Q}(\tau > t; \sigma) = \mathbb{E}\left[\mathbf{1}_{\{N_t^Y > b\}}\right], \tag{46}$$

where $N_t^Y := \min_{0\leq s\leq t} Y_s$.

Next, the following function is defined:

$$g(b,t) := \mathbb{Q}(\tau > t) - \mathbb{Q}(\tau > t; \sigma) \tag{47}$$

Then we consider the Fourier transform of the function $e^{\alpha b} g(b,t)$:

$$\begin{aligned}
&\int_{\mathbf{R}} e^{iub} e^{\alpha b} g(b,t) db \\
&= \int_{\mathbf{R}} e^{iub+\alpha b} \mathbb{E}\left[\mathbf{1}_{\{N_t^X > b\}} - \mathbf{1}_{\{N_t^Y > b\}}\right] db \\
&= \int_{\mathbf{R}} e^{iub+\alpha b} \int_{\mathbf{R}} \left(\mathbf{1}_{\{y>b\}} \rho_{N_t^X}(y) - \mathbf{1}_{\{y>b\}} \rho_{N_t^Y}(y)\right) dy db \\
&= \int_{\mathbf{R}} dy \rho_{N_t^X}(y) \int_{-\infty}^{y} e^{iub+\alpha b} db - \int_{\mathbf{R}} dy \rho_{N_t^Y}(y) \int_{-\infty}^{y} e^{iub+\alpha b} db \\
&= \int_{\mathbf{R}} \frac{e^{(iu+\alpha)y}}{iu+\alpha} \rho_{N_t^X}(y) dy - \int_{\mathbf{R}} \frac{e^{(iu+\alpha)y}}{iu+\alpha} \rho_{N_t^Y}(y) dy \\
&= \frac{\Psi_{N_t^X}(u - i\alpha) - \Psi_{N_t^Y}(u - i\alpha)}{iu+\alpha},
\end{aligned} \tag{48}$$

where $\rho_Z(\cdot)$ and $\Psi_Z(\cdot)$ denote the density function and the characteristic function of the random vector Z respectively.

Let the function $\xi(q,u)$ denote the Laplace transform in t of (48).

$$\xi(q,u) = \int_0^{+\infty} e^{-qt} \int_{\mathbf{R}} e^{iub} e^{\alpha b} g(t,b) db dt. \tag{49}$$

By Lemma 2, the function $\xi(q,u)$ can be expressed as

$$\begin{aligned}\xi(q,u) &= \frac{1}{iu+\alpha} \\ &\times \left\{ \int_0^{+\infty} e^{-qt}\Psi_{N_t^X}(u-i\alpha)dt - \int_0^{+\infty} e^{-qt}\Psi_{N_t^Y}(u-i\alpha)dt \right\} \\ &= \frac{1}{q(iu+\alpha)} \left\{ \Phi^-_{q,X}(u-i\alpha) - \Phi^-_{q,Y}(u-i\alpha) \right\},\end{aligned} \tag{50}$$

Thus, we can obtain the function $e^{\alpha b}g(t,b)$ by inverting the Fourier transform and the Laplace transform:

$$e^{\alpha b}g(t,b) = \frac{1}{2\pi i}\int_{\varsigma-i\infty}^{\varsigma+i\infty} e^{qt}\frac{1}{2\pi}\int_{\mathbf{R}} e^{-iub}\xi(q,u)dudq. \tag{51}$$

Applying Lemma 3 for (51), we complete the proof of Theorem 2.

□

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